

THE COLLEGE FINALS



The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a

INTEGRAL #1

**READY,
GET SET,...**

2:30

INTEGRAL #1

$$\int \frac{x}{e^{x^2}} dx$$

INTEGRAL #1

$$\int \frac{x}{e^{x^2}} dx$$

$$= \int x e^{-x^2} dx$$

$$= -\frac{1}{2} \int e^u du \quad u = -x^2, \quad du = -2x dx$$

$$= -\frac{e^u}{2} + C =$$

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INTEGRAL #2

**READY,
GET SET,...**

2:30

INTEGRAL #2

$$\int \frac{x}{x^4 + 6x^2 + 9} dx$$

INTEGRAL #2

$$\int \frac{x}{x^4 + 6x^2 + 9} dx$$

$$= \int \frac{x}{(x^2 + 3)^2} dx$$

$$= \frac{1}{2} \int \frac{1}{u^2} du \quad u = x^2 + 3, \quad du = 2x dx$$

$$= \frac{1}{2} \cdot \frac{-1}{u} + C = \frac{1}{2(x^2 + 3)} + C$$

INTEGRAL #3

**READY,
GET SET,...**

2:30

INTEGRAL #3

$$\int \frac{x + 4}{x^2 + 4} dx$$

INTEGRAL #3

$$\int \frac{x + 4}{x^2 + 4} dx$$

$$= \int \frac{x}{x^2 + 4} + \frac{4}{x^2 + 4} dx$$

$$= \frac{1}{2} \ln(x^2 + 4) + 2 \arctan \frac{x}{2} + C$$

INTEGRAL #4

**READY,
GET SET,...**

2:30

INTEGRAL #4

$$\int \frac{\ln x}{\sqrt{x}} dx$$

INTEGRAL #4

$$\int \frac{\ln x}{\sqrt{x}} dx$$

$$= 2\sqrt{x} \ln x - \int 2\sqrt{x} \cdot \frac{1}{x} dx \quad \text{by parts: } u = \ln x, \quad dv = \frac{dx}{\sqrt{x}}$$

$$= 2\sqrt{x} \ln x - 2 \int \frac{1}{\sqrt{x}} dx$$

$$= 2\sqrt{x} \ln x - 4\sqrt{x} + C$$

INTEGRAL #5

**READY,
GET SET,...**

2:30

INTEGRAL #5

$$\int \frac{\sqrt{x}}{x\sqrt{x} + \sqrt{7}} dx$$

INTEGRAL #5

$$\int \frac{\sqrt{x}}{x\sqrt{x} + \sqrt{7}} dx$$

$$= \frac{2}{3} \int \frac{1}{\sqrt{u}} du \quad u = x\sqrt{x} + 7, \quad du = \frac{3}{2}\sqrt{x}$$

$$= \frac{2}{3} \cdot 2\sqrt{u} = \frac{4}{3} \sqrt{x\sqrt{x} + 7}$$

INTEGRAL #6

**READY,
GET SET,...**

2:30

INTEGRAL #6

$$\int \frac{x^2}{x^2 + 1} dx$$



INTEGRAL #7

INTEGRAL #7

$$\int x \sec x \tan x \, dx$$

INTEGRAL #7

$$\int x \sec x \tan x \, dx$$

integration by parts: $u = x$ $dv = \sec x \tan x \, dx$
 $du = dx$ $v = \sec x$

$$= x \sec x - \int \sec x \, dx$$

$$= x \sec x - \ln|\sec x + \tan x| + C$$



INTEGRAL #8

$$\int x^p \frac{1}{x+5} dx$$

INTEGRAL #8

$$\int x^p \frac{1}{x+5} dx$$

$$= \int (u-5)\sqrt{u} du \quad u = x+5, \quad du = dx$$

$$= \int u^{3/2} - 5u^{1/2} du = \frac{2u^{5/2}}{5} - 5 \cdot \frac{2u^{3/2}}{3} + C$$

$$= \frac{2(x+5)^{5/2}}{5} - \frac{10(x+5)^{3/2}}{3} + C$$

INTEGRAL #9

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**READY,
GET SET,...**

2:30

INTEGRAL #9

$$\int (\sec x + \tan x)^2 dx$$

INTEGRAL #9

$$\int (\sec x + \tan x)^2 dx$$

$$= \int \sec^2 x + 2 \sec x \tan x + \tan^2 x \, dx$$

$$= \int \sec^2 x + 2 \sec x \tan x + \sec^2 x - 1 \, dx$$

$$= \int 2 \sec^2 x + 2 \sec x \tan x - 1 \, dx$$

$$= 2 \tan x + 2 \sec x - x + C$$

INTEGRAL #10

**READY,
GET SET,...**

2:30

INTEGRAL #10

$$\int \frac{dx}{x^2 + 2x + 10}$$

INTEGRAL #10

$$\int \frac{dx}{x^2 + 2x + 10}$$

$$= \int \frac{dx}{(x + 1)^2 + 9}$$

$$= \int \frac{du}{u^2 + 3^2} \quad u = x + 1, \quad du = dx$$

$$= \frac{1}{3} \arctan \frac{x+1}{3}$$

INTEGRAL #11

**READY,
GET SET,...**

2:30

INTEGRAL #11

$$\int \frac{dx}{\sqrt{x+1} + \sqrt{x-1}}$$

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$$\int \frac{1}{\sqrt{x}}$$

$$= \int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} dx$$

$$= \int$$

$$= \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + C$$

INTEGRAL #12

**READY,
GET SET,...**

2:30

INTEGRAL #12

$$\int \arctan \frac{1}{x} dx$$

INTEGRAL #12

$$\int \arctan \frac{1}{x} dx$$

integrate by parts: $u = \arctan \frac{1}{x}$ $dv = dx$ #
 $du = -\frac{1}{x^2+1} dx$, $v = x$

$$= x \arctan \frac{1}{x} + \int \frac{x}{x^2 + 1} dx$$

$$= x \arctan \frac{1}{x} + \frac{1}{2} \ln(x^2 + 1) + C$$