HIGH SCHOOL ROUND ONE

You will have ¨±o i `k¬¨P¡ to evaluate each of the fifteen definite integrals that will displayed one at a time on this screen. 9ff :k¡ ±P~¡ i ¬¡¨ AP ¡` i rf`¼PMv At the end of the two minutes, all hands must go up and judges will grade your answers immediately. For each correct answer, you will receive one ra e ticket to be entered for prizes that will be drawn after dinner.

At most five participants will move to the Finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary. O¯P~µokP i o¯`kY ¨o ¨]P T`k:f¡ ±`ff ~PIP`¯P NªVv

INTEGRAL #1

INTEGRAL #1

$$\int_{0}^{=2} (2\sin x + 3\cos x)\ dx$$

2:00

INTEGRAL #1

$$\int_0^{=2} (2\sin x + 3\cos x)\, dx$$

$$= \left[-2\cos \quad x \right.$$

INTEGRAL #2

READY,
GET SET,...

2:00

INTEGRAL #2

$$\int_1^2 x^2 \left(x - \frac{1}{x} \right) \, dx$$

2:00

INTEGRAL #2

$$\int_1^2 x^2 \left(x - \frac{1}{x} \right) \, dx$$

$$= \int_1^2 (x^3 - x) \, dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_1^2$$

$$= \boxed{\frac{9}{4}}$$

INTEGRAL #3

READY,

GET SET,...

2:00

INTEGRAL #3

$$\int_{-1}^{1} \sqrt[3]{\frac{x+1}{2}}\, dx$$

2:00

INTEGRAL #3

$$\int_{-1}^{1} \sqrt[3]{\frac{x+1}{2}}\, dx$$

$$= 2\int_{0}^{1} \sqrt[3]{u}\, du \qquad \left[u = \frac{x+1}{2}; \quad du = \frac{1}{2}\, dx \right]$$

$$= 2\left[\frac{3u^{4=3}}{4} \right]_{0}^{1}$$

$$= \boxed{\frac{3}{2}}$$

INTEGRAL #4

READY,
GET SET,...

2:00

INTEGRAL # 4

$$\int_{0}^{=4} \tan^2 x \sec^2 = 4 \tan^2 0$$

2:00

INTEGRAL #4

$$\int_0^{=4} \tan^2 x \sec^2 x \, dx$$

$$= \int_0^1 u^2 \, du \qquad [\, u = \tan x; \qquad du = \sec^2 x \, dx \,]$$

$$= \left[\frac{u^3}{3} \right]_0^1$$

$$= \boxed{\frac{1}{3}}$$

2:00

INTEGRAL #5

$$\int_1^2 \frac{3x^4 + 5x^2 + 2}{x^2}\,dx$$

$$= \int_1^2 \left(\frac{3x^4}{x^2} + \frac{5x^2}{} \right.$$

INTEGRAL #6

READY,
GET SET,...

2:00

INTEGRAL #6

$$\int_{-1}^{0} (x+2)(x^2+4x+3)^2 \, dx$$

2:00

INTEG

$$\int_{-1}^{0} (x+2) \; 4x \; dx$$

$$\left[\, u = x^2 + 4x \quad = 2x + 4\, dx = 2(x+2)\, dx \,\right]$$

$$= \frac{1}{2}\int_{0}^{3} u^2$$

$$= \frac{1}{2}\left[\quad \frac{9}{2} \right.$$

INTEGRAL #7

READY,
GET SET,...

2:00

INTEGRAL #7

$$\int_{2=}^{3=} \frac{1}{x^2} \sin \frac{1}{x} \, dx$$

2:00

INTEGRAL #7

$$\int_{2=}^{3=} \frac{1}{x^2} \sin \frac{1}{x}\, dx$$

$$= -\int_{=2}^{=3} \sin u\, du \qquad \left[u = \frac{1}{x}; \quad du = -\frac{1}{x^2} dx \right]$$

$$= \Big[\cos u \Big]_{=2}^{=3}$$

$$=$$

INTEGRAL #8

READY,
GET SET,...

2:00

INTEGRAL #8

$$\int_1^9 \frac{(1-\sqrt{x})^3}{\sqrt{}}$$

2:00

INTEGRAL #8

$$\int_1^9 \frac{(1-\sqrt{x})^3}{\sqrt{x}}\,dx$$

$$= -2\int_0^{-2} u^3\,du \qquad \left[\, u = 1-\sqrt{x}; \quad du = -\frac{1}{2\sqrt{x}}\,dx \,\right]$$

$$= -2\left[\frac{u^4}{4}\right]_0^{-2}$$

$$= \boxed{-8}$$

INTEGRAL #9

READY,
GET SET,...

2:00

INTEGRAL #9

$$\int_0^{3\ =2} \cos x \sqrt{1+\sin x}\, dx$$

2:00

INTEGRAL #9

$$\int_0^{3=2} \cos x \sqrt{1+\sin x}\, dx$$

$$= \int_1^0 \sqrt{u}\, du \quad [\, u = 1 + \sin x; \quad du = \cos x\, dx \,]$$

$$= \left[\frac{2u^{3=2}}{3}\right]_1^0$$

$$= \boxed{-\frac{2}{3}}$$

INTEGRAL #10

READY,
GET SET,...

2:00

INTEGRAL #10

$$\int_0^1 xe^x\,dx$$

2:00

INTEGRAL #10

$$\int_0^1 xe^x\,dx$$

$$\left[\text{integrate by parts: } \begin{array}{rl} u &= x \\ du &= dx \end{array};\quad \begin{array}{rl} dv &= e^x\,dx \\ v &= e^x \end{array}\right]$$

$$= \left[xe^x\right]_0^1 - \int_0^1 e^x\,dx$$

$$= e - \left[e^x\right]_0^1 = \boxed{1}$$

INTEGRAL #11

INTEGRAL #11

$$\int_0^1 \left(\sqrt{x^3} + \sqrt[3]{x^2} + \sqrt[4]{x} \right) \, dx$$

2:00

INTEGRAL #11

$$\int_0^1 \left(\sqrt{x^3} + \sqrt[3]{x^2} + \sqrt[4]{x} \right) \, dx$$

$$= \int_0^1 \left(x^{3=2} + x^{2=3} + x^{1=4} \right) \, dx$$

$$= \left[\frac{2x^{5=2}}{5} + \frac{3x^{5=3}}{5} + \frac{4x^{5=4}}{5} \right]_0^1$$

$$= \boxed{\frac{9}{5}}$$

INTEGRAL #12

READY,
GET SET,...

2:00

INTEGRAL #12

$$\int_0^{=3} \frac{1}{\cos 2x + \sin^2 x}\, dx$$

2:00

INTEGRAL #13

READY,
GET SET,...

2:00

INTEGRAL #13

$$\int_1^2 x\sqrt[3]{x-1}\,dx$$

2:00

INTEGRAL #13

$$\int_1^2 x\sqrt[3]{x-1}\,dx$$

$$= \int_0^1 (u+1)u^{1=3}\,du \quad [\,u = x-1;\ x = u+1;\ du = dx\,]$$

$$= \int_0^1 \left(u^{4=3} + u^{1=3}\right)\,du$$

$$= \left[\frac{3u^{7=3}}{7} + \frac{3u^{4=3}}{4}\right]_0^1 = \boxed{\frac{33}{28}}$$

INTEGRAL #14

RE
C

INTEGRAL #14

$$\int_0^1 (x-1)(x+1)(x^2+1)(x^4+1)\,dx$$

2:00

INTEGRAL #14

$$\int_0^1 (x-1)(x+1)(x^2+1)(x^4+1)\,dx$$

$$= \int_0^1 (x^2-1)(x^2+1)(x^4+1)\,dx$$

$$= \int_0^1 (x^4-1)(x^4+1)\,dx$$

$$= \int_0^1 (x^8-1)\,dx = \left[\frac{x^9}{9} - x\right]_0^1 = \boxed{-\frac{8}{9}}$$

INTEGRAL #15

READY,
GET SET,...

2:00

INTEGRAL #15

$$\int_0^{\ln 2} e^{2x}\sqrt{e^x - 1}\,dx$$

2:00

TEGRAL #15

$$\int_0^{\ln 2} e^{2x}\sqrt{e^x - 1}\,dx$$

$$\int_0^{\ln 2} e^x \cdot e^x\sqrt{e^x - 1}\,dx$$

$$[\, u = e^x - 1; \quad e^x = u + 1; \quad e^x\,dx = du \,]$$

$$= -\int_0^1 (u+1)\sqrt{u}\,du = -\left[\frac{2u^{5/2}}{\quad} + \frac{2u^{3/2}}{\quad}\right]_{\quad}^{1} - \frac{16}{\quad}$$