

# INTEGRATION BEE ROUND ONE: DEFINITE INTEGRALS

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You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen.

At the end of the two minutes, all hands must go up and judges will grade your answers immediately. If you wish to protest an answer, notify a grader before the next integral is displayed.

For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.





# INTEGRATION BEE ROUND ONE: DEFINITE INTEGRALS

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- You do **not** need to combine multiple logs or fractions with irrational numbers (e.g.  $\pi$ ,  $e$ ,  $\ln 3$ ).

$$2 \ln 3 \checkmark, \ln 9 \checkmark$$

$$\frac{1}{e} + 2 \checkmark, \frac{1 + 2e}{e} \checkmark$$

- However, **not** a negative powers should be

READY,  
GET SET,...

2:00

INTEGRAL # 1

$$\int_1^2 \frac{20x + 24}{x^3} dx$$

## INTEGRAL # 1

$$\int_1^2 \frac{20x + 24}{x^3} dx$$

$$= \int_1^2 (20x^{-2} + 24x^{-3}) dx$$

$$= \left[ \frac{20}{x} + \frac{12}{x^2} \right]_1^2$$

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## INTEGRAL # 2

$$\int_1^4 \frac{6}{x(1+x)^3} dx$$



## INTEGRAL # 2

$$\int_1^4 \frac{6}{x(1+\sqrt{x})^3} dx$$

$$= 12 \int_2^3 u^{-3} du \quad u = 1 + \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}} dx$$

$$= 6 u^{-2} \Big|_2^3$$

$$= 6 \left( \frac{1}{9} - \frac{1}{4} \right) = -\frac{5}{6}$$

$$= \boxed{\frac{5}{6}}$$



INTEGRAL # 3

$$\int_1^2 e^{3 \ln x^2} 2 \ln x \, dx$$

### INTEGRAL # 3

$$\begin{aligned}
 & \int_1^2 e^{3\ln x^2 - 2\ln x} dx \\
 &= \int_1^2 e^{\ln x^6 - \ln x^2} dx \\
 &= \int_1^2 e^{\ln x^4} dx \\
 &= \int_1^2 x^4 dx \\
 &= \frac{x^5}{5} \Big|_1^2 = \frac{1}{5} (2^5 - 1) = \frac{31}{5} \text{ or } 6\frac{1}{5}
 \end{aligned}$$

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READY,  
GET SET,...

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## INTEGRAL # 5

$$\int_0^3 \frac{2x^3}{x^2 + 3x + 18} dx$$

## INTEGRAL # 5

$$\int_0^3 \frac{2x + 3}{x^2 + 3x + 18} dx$$

$$= \int_{18}^{18} \frac{1}{u} du \quad u = x^2 + 3x + 18, \quad du = (2x + 3) dx$$

$$=$$

READY,  
GET SET,...

2:00

## INTEGRAL # 6

$$\int_1^{e^4} \frac{(\ln x)^2}{x} dx$$

## INTEGRAL # 6

$$\int_1^{e^4} \frac{(\ln x)^2}{x} dx$$

$$= \int_0^4 u^2 du \quad u = \ln x, \quad du = \frac{1}{x} dx$$

$$= \frac{u^3}{3} \Big|_0^4$$

$$= \frac{64}{3} \text{ or } 21\frac{1}{3}$$



## INTEGRAL # 7

 $\int_1^1$ 

$$18x^5 + 3x^2 + x \, dx$$

 $1$

## INTEGRAL # 7

 $Z_1$ 

$$\int_1^3 (18x^5 + 3x^2 + x) dx$$

$$= 3x^6 + x^3 + \frac{x^2}{2} \Big|_1^3$$

$$= 3 + 1 + \frac{1}{2} \cdot 3^2 - \left( 3 + 1 + \frac{1}{2} \right)$$

$$= \boxed{2}$$





INTEGRAL # 8

$$\int_0^{\pi/6}$$

$$\sec 2x \tan 2x \, dx$$

0

## INTEGRAL # 8

$$\begin{aligned} & \int_0^{\pi/6} \sec 2x \tan 2x \, dx \\ &= \frac{1}{2} \int_0^{\pi/3} \sec u \tan u \, du \quad u = 2x, \quad du = 2 \, dx \\ &= \frac{1}{2} \left[ \sec u \right]_0^{\pi/3} \\ &= \frac{1}{2} (2 - 1) \\ &= \boxed{\frac{1}{2}} \end{aligned}$$

READY,  
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2:00

## INTEGRAL # 9

$$\int_1^1 \frac{x^2 + 1}{x^3 + 2x^2 + x + 2} dx$$



READY,  
GET SET,....

2:00









## INTEGRAL # 11

 $\int_0^{\pi}$  $(\sin x + \cos x)^2 dx$ 

0





## INTEGRAL # 12

$$\int_0^{\frac{1}{3}} \frac{1}{(1+2x)^2} dx$$

INTEGRAL # 12

$$\begin{aligned}
 & \int_0^3 \frac{1}{(1+2x)^2} dx \\
 &= \frac{1}{2} \int_1^7 \frac{1}{u^2} du \quad u = 1 + 2x, \quad du = 2 dx \\
 &= \frac{1}{2} \int_1^7 u^{-2/3} du \\
 &= \frac{1}{2} \left[ 3u^{1/3} \right]_1^7 = \frac{3}{2}(3 - 1) = \boxed{3}
 \end{aligned}$$

READY,  
GET SET,...

2:00







READY,  
GET SET,...

2:00

## INTEGRAL # 14

$$\int_0^{\pi/2} \sin x \sqrt{1 + 3 \cos x} \, dx$$

INTEGRAL # 14

$$\int_0^{\pi/2} \frac{1}{\sin x \sqrt{1 + 3 \cos x}} dx$$

$$= \frac{1}{3} \int_4^1 \frac{1}{\sqrt{u}} du \quad u = 1 + 3 \cos x, \quad du = -3 \sin x dx$$

$$= \frac{1}{3} \int_1^4 \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{3} \left[ \frac{2}{3} u^{3/2} \right]_1^4$$

$$= \frac{2}{9} (8 - 1) = \frac{14}{9} \text{ or } 1\frac{5}{9}$$

READY,  
GET SET,...

2:00

## INTEGRAL # 15

$$\int_2^3 \frac{x+5}{(4-x)^4} dx$$

INTEGRAL # 15

$$\begin{aligned}
 & \int_2^3 \frac{x+5}{(4-x)^4} dx \\
 &= \int_2^1 \frac{4-u+5}{u^4} du \quad u=4-x, \quad x=4-u, \quad du=-dx \\
 &= \int_2^1 u^{-3} - 9u^{-4} du \\
 &= \left[ \frac{1}{2} u^{-2} - \frac{9}{3} u^{-3} \right]_2^1 \\
 &= \left[ \frac{1}{2} - \frac{9}{4} \right] - \left[ \frac{1}{8} - \frac{27}{8} \right] = \frac{9}{4} - \frac{1}{4} = \frac{8}{4} = 2
 \end{aligned}$$